

Maria's Mixed-up Pasta

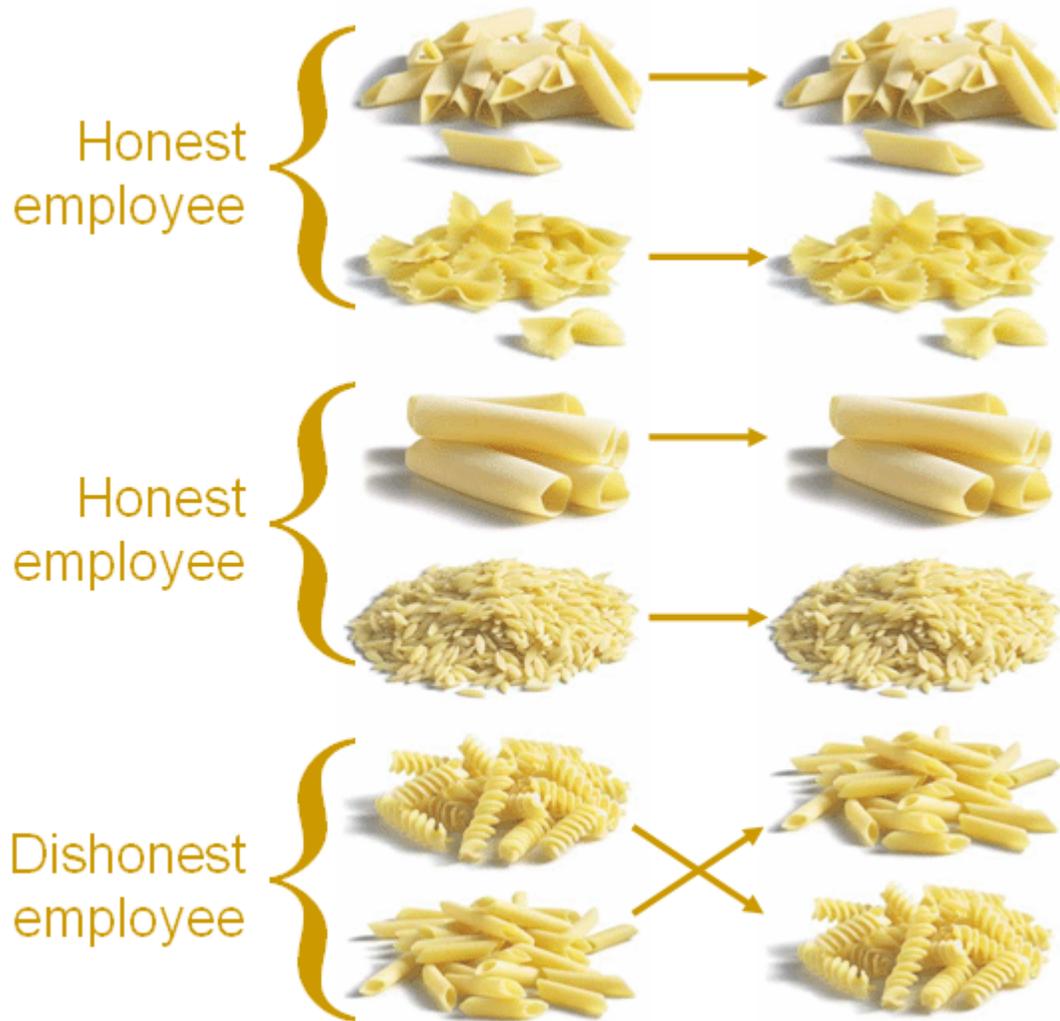
Once upon a time there was an Italian village whose only attraction was *Maria the Magnificent*. She was not called magnificent because she looked magnificently attractive, but because she made dishes that were magnificently tasty. Every Friday she ordered sacks of cannelloni, fusilli, penne, and farfalloni for the next week, and every Monday morning, the horse and cart from the big city would come into the village courtyard and drop off the pasta sacks on the terracotta tiles outside her restaurant. As a joke, the pasta maker, Giovanni, always switched the labels on the four bags, so that none of them was ever labeled correctly. For example, the first time she ordered, the cannelloni (upper left) was labeled fusilli (lower right), the fusilli was labeled farfalloni (lower left), the farfalloni was labeled penne (upper right), and the penne was labeled cannelloni.



How many sacks did Maria the Magnificent have to inspect, in order to re-label all of the sacks correctly?

Extensions:

- One day the supplier wrote a note along with the 4 pasta sacks. "My dear Maria, this week one of your sacks are correctly labeled. I am training my son Luigi, and he is too young, too honest and doesn't understand our joke. The other sacks have their labels switched. Sincerely, Giovanni." How many sacks did Maria the Magnificent have to inspect, to do the relabeling?
- The next week, Giovanni wrote again, informing Maria the magnificent that from now on honest Luigi would pack and label two of her sacks. Under this spate of honesty, how many sacks did Maria the Magnificent have to inspect, on average?
- Luigi soon decided to go out and start his own rival pasta business. He hires two honest labelers and by mistake one dishonest labeler. When Maria the Magnificent's business expands to a six pasta menu, Luigi sends six sacks of pasta; 2 labeled by each of his 3 employees. Giovanni hires three dishonest labelers and sends Maria the Magnificent six sacks of pasta; 2 labeled by each of his 3 employees. All dishonest employees switch the labels on their two sacks.



An example of the labeling done by Luigi's Employees.

Each sack's contents is on the left and its picture-label is on the right.

- In order for Maria the Magnificent to correctly redistribute the labels, how many of Luigi's sacks does she need to open on average? How many of Giovanni's sacks?
- Umberto, another of Giovanni's customers ordered his *three-packet penne-fusilli pasta sample*. Giovanni stuffed three packets as follows:
 - Packet A: 2 penne
 - Packet B: 2 fusilli
 - Packet C: 1 penne and 1 fusilli

Of course all the labels were wrong. Giovanni wrote: "Umberto, if you open any packet you will only be able to see the top piece of pasta. What is the fewest number of packets you have to open in order to correctly fix my labeling mistake ;-). Sincerely, Giovanni."



- After a year, Maria the Magnificent decided to add a seventh pasta, rigattonici, to her menu.



- The seven sacks of pasta arrived with a note from Giovanni: "Dear Maria, I packed and labeled all your sacks this time ;-)" How many sacks did Maria the Magnificent have to inspect in order to re-label all of the sacks correctly?
- Create your own problem based on Luigi, Giovanni and the seven sacks of pasta.

The Math in This Problem:

This logic puzzle involves studying the various arrangements of correctly-labelled and mislabelled pasta sacks. Progressing through this brainteaser will provide students with an understanding of permutations, which is an area of mathematics associated with the rearrangement of multiple objects or values.

