# LEARNING MATHEMATICS IN AN ACCESSIBLE CLASSROOM

**Research Report**

**Executive Summary**

**by**

**Dr. Sharon Friesen**

**Dr. Pat Clifford**

**with**

**Krista Francis-Poscente**

**Barbara Martin**

**Galileo Educational Network**
**University of Calgary**

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## Executive Summary

### Context

"Today, mathematics education faces two major challenges: raising the floor by expanding achievement for all, and lifting the ceiling of achievement to better prepare future leaders in mathematics, as well as in science, engineering, and technology. At first glance, these appear to be mutually exclusive" (Research Points, 2006, p.1). But are they? Is it possible to design learning that engages the vast majority of students in higher mathematics learning?

The purpose of this study is to determine whether the principles of Universal Design for Learning (UDL) result in increased student mathematical proficiency and achievement for all students in a Grade 7 classroom, including those with identified learning needs.

Alberta students consistently score very well on international (PISA, TIMSS) and national (SAIP) mathematics studies[1]. Given such high international and national standings, many might question why Alberta Education would be interested in ensuring even higher achievement for all students in the area of mathematics. Perhaps this can be best explained by a brief conversation that Dr. Friesen had with an individual from Alberta Education's Assessment Branch. In discussing Alberta's success on the recently released PISA 2006 findings, in which Alberta scored second only to Finland, this person stated, "We still have work to do. There is no place to stand still. If you are standing still you are actually going backwards."

This research study is designed to encourage continued conversation about going forward with mathematics education in this province, particularly in terms of:

- better meeting the needs of Alberta's increasingly diverse student population;
- reducing the number of students who give up on the study of mathematics;

[1] See reports http://education.alberta.ca/admin/testing/nationaltesting.aspx

**Building On Previous Alberta Research Studies**
In 2006, Alberta Education contracted Dr. Sharon Friesen from the Galileo Educational Network to conduct a research study to:

- identify and describe an innovative, accessible classroom;
- describe how digital technologies are and might be used to enable all learners. These technologies include devices, media and services currently on computers or those that could be incorporated to ensure all students are equitably engaged in learning;
- identify and describe the ways in which a teacher uses or might use digital technologies to extend and enrich learning for all students in the regular classroom;
- envision what might be possible in creating an accessible classroom;
- design what an optimal accessible classroom might look like,
- provide recommendations for teachers, schools, school districts and governments on the creation of accessible classrooms; and
- add to the body of research knowledge and theory about the factors that contribute to the successful accessible classroom

The 2006 study found that:

- accessible classrooms are media rich;
- accessible classrooms follow the principles of Universal Design for Learning;
- teachers of accessible classrooms make the curriculum accessible to all learners; and
- accessible classrooms require learning focused networks.

In 2007 Alberta Education contracted Dr. Friesen to conduct a second study to build on findings from 2006. Of particular interest were ways that the four findings would play themselves out in a mathematics classroom.

Results are reported in this document, *Learning Mathematics in an Accessible Classroom.*

**Goals and Purpose**
The purpose of the research was to investigate and report on:

- the impact of Universal Design for Learning on student mathematics proficiency and achievement
- instructional practices that support mathematics learning for all students, particularly those who are identified with special needs.

The goals of the research were:

- to determine the academic achievement of a diverse group of students in a Grade 7 mathematics classroom through a statistically valid and reliable pretest;

- to determine the look and feel for the context of the classroom through videotaping;
- to work with the classroom teacher to design a study based on the principles of Universal Design for Learning;
- to determine the academic achievement of the same group of students through a statistically valid and reliable post-test;
- to provide a visual image of a mathematics classroom that follows the principles of Universal Design for Learning; and
- to add to the research body on how to create effective learning environments for diverse learners.

**Design of the Study**

The researchers chose design-based research to accomplish these goals. “Design-based research can help create and extend knowledge about developing, enacting, and sustaining innovative learning environments” (The Design-Based Research Collective, 2003, p.5).

Design-based research is particularly sensitive to local contexts. The designed intervention, a unit on Grade 7 Geometry, was created with the certainty that how students actually responded would change subsequent lessons and activities. That is, there was no attempt to design or implement material or activities created outside the context of this classroom, and these students.

Our challenge was “to develop methodologies which recognize complexities and yet produce robust measures of impact or added value” in order to contribute to the understanding of policy makers (Pittard, 2004, p.181). We did this not by designing a program that could be scaled for delivery across the province. Rather, we extracted examples and principles for responsive teaching in technology-rich environments that improve mathematical proficiency and achievement for all students, including those identified with special learning needs.

Both quantitative and qualitative methodologies were employed throughout.

**UDL Design Principles In The Mathematics Classroom**

The pervasiveness of the North American script for teaching mathematics provides essential context as we start to focus on the potential for UDL in the mathematics classroom. If fundamental principles of UDL become tied to a teaching script focused on practicing routine, procedural exercises, then mathematical proficiency for all students will not become a reality. Designing mathematics learning for the UDL classroom and teaching mathematics in a UDL classroom suddenly get far more complicated than first imagined, for now it is not just the principles of UDL that need to be brought to bear, but also what research tells us about gaining mathematical proficiency.

Teaching for mathematical proficiency (i.e., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and a productive disposition) requires that the teachers design a learning environment that provides “a solid foundation of detailed knowledge and clarity about the core concepts around which that knowledge is organized to support effective learning” (Donovan and Bransford, 2005, p.569). The type of practice required to promote mathematical proficiency stands in sharp contrast both to transmission-type pedagogies and to discovery-type pedagogies. Rather, the type of practice that builds mathematical proficiency requires that students be brought into a collaborative “relationship between different facts students are learning, between the procedures they are learning, and the underlying concepts” through robust, rich problems and investigations (Shanker Institute, 2005, p.7).

It is to this type of mathematical learning environment that the principles of UDL need to be tethered.

## Summary of Findings

1. **All students showed significant improvement in achievement.**
   The PISA test items used for pre and post-tests were chosen (1) for their validity and reliability and (2) for their ability to measure mathematical proficiency. The four items had levels of difficulty from middle to highest range. The instructional intervention was not designed to “teach to the test”. Rather, all elements were designed to build mathematical proficiency that would transfer to a number of contexts, one of which is standardized testing of the highest international caliber.

   Common sense worries about changing mathematics instruction to better meet the needs of special needs students were not realized. All students improved on all items. Mean scores for all tasks demonstrate statistically significant improvement for coded LD students, for not-coded LD students and for the class as a whole. Thus, it is possible to raise both the ceiling and the floor of student achievement by incorporating UDL principles into the design of mathematics curricula.

2. **All students demonstrated gains in the five strands of mathematical proficiency.**
   Kilpatrick, Swafford and Findell (2001) define mathematical proficiency in terms of five intertwining strands:
   - *conceptual understanding* – an understanding of concepts, operations and relations. Conceptual understanding frequently results in students’ comprehending connections and similarities among interrelated facts.
   - *procedural fluency* – flexibility, accuracy and efficiency in implementing

appropriate procedures. Skill in proficiency includes the knowledge of when and how to use procedures. This includes efficiency and accuracy in basic computations.

- *strategic competence* – the ability to formulate, represent and solve mathematical problems. This is similar to problem solving. Strategic competence, conceptual understanding and procedural fluency are mutually supportive.
- *adaptive reasoning* - the capacity to think logically about concepts and conceptual relationships. Reasoning is needed to navigate through the various procedures, facts and concepts required to arrive at solutions.
- *productive disposition* – positive perceptions about mathematics. Productive disposition develops as students gain more mathematical understanding and become capable of learning and doing mathematics.

Analysis of the qualitative data demonstrates the developing mathematical proficiency of students in this Grade 7 classroom as evidenced in their ability to dialogue with each other, to explore concepts in depth, to think and reason, to test conjectures and justify solutions.

When considering the power of UDL principles to change the dominant procedural script of mathematics teaching, it is especially important to note that the instructional intervention involved five essential and connected elements: (1) mathematical content knowledge; (2) pedagogical content knowledge for mathematics; (3) UDL principles; (4) assessment for learning and (5) an instructional design process that supports the effective integration of mathematics strands as identified in the Program of Studies.

3. **All students can engage with difficult mathematical ideas when they are provided with dynamic assessment.**

Assessment for learning places teachers and students in a design environment in which constant feedback informs next teaching and learning steps. As Black (2004) indicates, there is a great deal of confusion about the kinds of assessment that builds proficiency and improves achievement. In this study, students received dynamic feedback in a number of ways:

- From teachers, in response to their individual work
- From teachers, in response to the emergent design of lessons and activities to address misconceptions
- From peers as they worked and talked together
- From the learning environment, particularly in the case of the dynamic geometry software

It is important to emphasize the difference between dynamic assessment and feedback through tests, quizzes and assignments designed for purposes of accountability, ranking of students, or certifying competence. The latter assessment practices are particularly damaging to students "with low attainments who are led to believe they lack 'ability' and are not able to learn"(Black, 2004, p.1).

Learning goals remained the same for all students throughout the study. What changed was instructional design that included multiple means of representation and expression. When (1) the learning task was mathematically robust; (2) the representation of concepts was varied in pedagogically sound ways and (3) students were given a range of opportunities to express their emerging understandings, then all students were able to engage deeply. They volunteered their attention to and interest in the learning task.

4. **The principles of UDL permit teachers to break the stranglehold of the procedural script for teaching mathematics.**
   Creating more robust and interesting mathematical tasks, problems or inquiries is a necessary component of the design for accessible classrooms. However, it is not sufficient to provide more robust, complex problems intended to create mathematical proficiency (Stigler and Hiebert, 1999; Shanker Institute, 2005). The dominant North American script for teaching mathematics is so ingrained that teachers turned even the best problems into routine, procedural exercises.

   Incorporating UDL principles into instructional design has the potential to change instruction at its root, disrupting the processes by which many students come to be labeled as unable to learn mathematics.

5. **Access to technology is a critical factor in an accessible mathematics classroom.**
   Currently, the use of technology in UDL emphasizes the role of assistive technologies that permit students with identified needs to adapt to the pervasively print environment of most classrooms. AT has a definite role to play in creating more accessible learning opportunities for all students.

   However, AT alone may leave untouched the procedural script for teaching mathematics if it leaves assumptions about the effective development of mathematical proficiency unchallenged. We can easily imagine classrooms in which, for examples, technology is introduced so that weaker students can in some sense keep up with the demands of fast-right-answer-giving, or where modifications that “dumb down” or fragment experiences are provided in the name of assistance.

What this study demonstrates is that the inherent nature of digital environments such as Geometer’s Sketchpad™ and IO to represent and express mathematical concepts in dynamic ways.

6. **Introducing UDL into the mathematics classroom is a disruptive innovation.**

While the goal of creating increasingly accessible classrooms seems incontrovertible, actually creating the changes that make a difference for students disrupts the status quo.

(1) The research team made a number of other attempts to introduce range of technology-rich environments: email, wiki and access to a common drive and print-outs of emerging work. Difficulties with district policies and the school network and resources limited our opportunities to do this. As a work-around, we created an online website so that students could access Geometer’s Sketchpad™ activities and instructions.

The school jurisdiction had made considerable effort to respond to findings of Friesen’s 2006 report on accessible classrooms. On short notice, software was installed, enabled email and ensured that Universal Access features were accessible from all desktops. We appreciated these efforts, and could not have conducted the study without them. Nor could we have designed instruction incorporating principles of UDL had students not had access to laptop computers that functioned well.

In this report we note areas in which improvement is still possible.
(2) Rigid timetables, a ubiquitous feature of all secondary schools, interfere with the capacity of students to engage with learning in ways that build mathematical proficiency. When daily work is fragmented into short blocks of time, students and teachers become frustrated by arbitrary (and in our view, unnecessary) constraints on engagement.

Block timetables have a checkered history in secondary school reform. Unsuccessful attempts to introduce reform by increasing class times to 90 or 120 minute blocks without changing the teaching script, itself, have left both students and teachers frustrated. Doing more of the same kinds of procedural exercises, now for double or triple the time becomes excruciating. As with other elements in this study, we emphasize that a structural change, alone, will not make the kinds of difference we report here.

However, the teacher and student responses to constraints of the timetable confirm what we have found in other contexts. When students become engaged in the ways described here, both they and their teachers demand longer blocks of uninterrupted time for their work. It is our experience that introducing this kind of innovation inevitably puts pressure on existing structures such as the timetable.

(3) We had not anticipated the extent to which the increased proficiency of coded LD students disrupted the social hierarchies of the classroom. Students who considered themselves (or were considered by others) to be better at math were initially very uncomfortable with the emerging confidence and ability of students they thought were less able.

Disruptions of this sort point, perhaps, to the tenacity of conventional teaching scripts. When teachers and students experience initial discomfort at the introduction of innovation, it is tempting to retreat to familiar ground.

It is easy to pen the words that describe access for all to high levels of mathematical proficiency. It will be more challenging to live with the inevitable pressures that such a goal will place on taken-for-granted, everyday structures and experiences.

7. **Creating accessible mathematics classrooms consistent with UDL principles requires increased teacher knowledge and support for on-going professional development.**
   Changing teaching practices and school, jurisdiction and classroom structures will require significant investment in professional development.

   (1) Most teachers, principals and senior administrators recognize the experience described by Mrs. Jamieson. They, themselves, have had unfortunate experiences with math in school—or they know many people in the same boat. Teaching mathematics that incorporates UDL principles requires teachers to design learning experiences in mathematics that they, themselves, have never experienced.

   Progress will require the active engagement of mathematicians and math educators to design pedagogical content knowledge that is mathematically sound. More math courses of the procedural sort will not get teachers out of their current dilemma. While most need more mathematics, it is mathematics of a particular sort: the kind that permits them to design instruction that gives students access to complex ideas.

   Mrs. Jamieson reported to the research team that she had followed up her involvement in the study with a summer course in mathematics. Knowing that she, herself, needed a deeper understanding of mathematical concepts, she was also clear that a course, alone, was unlikely to help in the ways she desired. "I want to be able to think like you," she told Dr. Friesen. And to do that, she knew she wanted more opportunities to explore pedagogical issues at the same time.

(2) Leadership to support teachers like Mrs. Jamieson, to provide them with useful feedback on their teaching for professional growth, and to make sound judgments about administrative issues such as timetables and allocation of resources requires a degree of understanding of UDL principles and mathematics that principals generally lack at this time. Developing the capacity to lead for learning of this sort cannot be left to chance.

## Recommendations

### 1. Create a curriculum for mathematics that draws upon the principles of UDL.

**Context**

Mathematics has several key elements for curriculum design using UDL strongly in place:

- Organizations such as the Pacific Institute for Mathematical Sciences (PIMS) have already indicated a strong desire to bring mathematicians, math educators and teachers to together to create robust problems and instructional design that increases math proficiency in both teachers and students.
- PIMS has already created a network of mathematicians, math educators, teachers and First Nations and Metis Elders to address the particular concerns of mathematics and First Nations and Metis students. This demonstrates PIMS willingness and capacity to address the issue of making mathematics accessible to all.
- The National Science Foundation has invested heavily in on-going work to create mathematically robust and engaging problems available at no cost to teachers.
- There is a developing history of professional development through Lesson Study in Canada and the US which involves mathematicians, math educators and teachers.
- The International Assessment Consortium from UK continue to identify the problems in practice with assessment—particularly struggles teachers have to build assessment for learning into their practice. We can build upon and contribute to this work.
- Alberta Education has a strong interest in exploring the application of UDL in general and in mathematics in particular.

That is, key elements of designing effectively for UDL in the mathematics classroom are already in place in other contexts. Alberta Education could draw quickly upon these resources to create an Alberta-made approach to the creation of accessible classrooms in mathematics.

The successful creation of this Alberta solution to the problem of raising the ceiling and lifting the floor could provide a model for changes to all subject areas.

**Implications for Alberta Education**

- Look for synergy partners like PIMS, the International Assessment Consortium and CAST who understand the particular issues of teaching mathematics, assessment for learning and UDL.
- Special Programs Branch should take the lead in bringing partners together to create a mathematics curriculum (understood in its broadest sense) designed according to the principles of UDL.
- Special Programs Branch should take the lead in developing and publishing resources that represent both mathematics and pedagogical content in multiple, flexible and technologically sophisticated ways.

**Implications for Universities**

- PIMS mathematicians and math educators come from the universities. Their involvement in creating a provincial curriculum is essential. It is also hoped that their involvement in this project would increase the effectiveness of teacher preparation in mathematics.
- Faculties of education must address the development of proficiency in all students, not as a special education topic, but as an integral part of their curriculum and instruction courses.

**2. Establish a network of teachers who are willing to form a Community of Practice.**

**Context**

Conventionally, new curricula are developed by some and delivered by others. In the U.S. we have seen the failure of this approach, even to the creation and dissemination of mathematically robust problems. Recommendation #2 suggests that the development of a mathematics curriculum based on UDL will require design research in which teachers are involved from the outset in multiple ways: in dialogue with mathematicians and math educators; in working through robust problems to increase their own mathematical understandings; to dialogue as they work in their classrooms; and to make their practice public so that others in the network can build their own mathematical and pedagogical proficiency. In essence, we are suggesting a new approach to developing curriculum by prototyping the innovation as it is being created.

In this report, we have suggested the potential pitfalls of attaching UDL principles to tenacious procedural scripts for the teaching of mathematics. It is easy to read about such principles and quickly assume that one knows how to teach with them. We anticipate, for example, educators who will dismiss their power by saying, “They are just good practice. There’s nothing really new in all this.”

If that happens, then the province will suffer a rash of “multiples” stuck on to existing resources and procedures. We do not underestimate the danger of this, nor the care with which one must proceed to develop innovations that will actually take hold effectively.

The support and active involvement of teachers willing to do what Mrs. Jamieson did—to try unfamiliar approaches over an extended period of time—will be key to the innovation’s success.

Alberta has the technological broadband infrastructure through SuperNet to permit teachers to connect in both synchronous and asynchronous ways. The community of practice does not need to be geographically limited. In fact, in terms of addressing issues of diversity, the capacity to have teachers from across the province--rural, urban, First Nations--working on the same issues is essential.

**Implications for Alberta Education**

- Special Programs Branch should take the organizing lead in bringing this network together and supporting its work with funding and resources.
- Special Programs Branch should issue a request for proposals (RFP) to school jurisdictions to become part of this network. This will ensure that school jurisdictions get behind the initiative.
- Design the RFP to include stipulations for buy-out time for participating teachers. In the past, CANARIE-sponsored initiatives provided participating teachers with a day a week to devote appropriate time and attention.
- Establish a research committee to conduct design research on the work and outcomes of the network

**Implications for School Jurisdictions**

- Allocate resources to the initiative.
- Develop processes to feed emerging work from the network back into the jurisdiction to develop the capacity of others to work in these ways
- Provide and support the necessary technological infrastructure

**Implications for Principals**

- Develop the instructional leadership capacity to direct and supervise work at the school level. Few principals will have taught in these ways, and it cannot be assumed that they will be able to give the most useful feedback possible when teachers introduce the innovation in their classrooms. It would do a disservice to principals and to teachers to establish a myth that UDL principles are just like all the other good things they have always done. Leaders must understand and be able to act on the differences that make a difference.
- Disruptions to the status quo are bound to occur. Of necessity, for example,
    - the need for new timetables may emerge.
    - understanding the dynamics of anticipated and unanticipated resistance that puts pressure on teachers to revert to conventional practices.

**Implications for Teachers**

- Active participation in a design research network will take time for participating teachers. It is unreasonable to ask people to do pioneering work without providing additional time and resources they find meaningful.
- Participants will be asked to demonstrate willingness to:
    - increase their own mathematical proficiency
    - learn the principles of UDL and understand their application to mathematics in particular
    - use technology both to represent concepts to students and to permit students to express knowledge in multiple ways
    - collaborate with others in ways that build new knowledge and "next practice"
    - make their practice increasingly public by sharing video clips; student responses to the work; struggles and successes in developing next practices, etc.