Road Side or Trailer Side?

As Jesse and I were mucking about near the sharply meandering creek on our property–walking alongside, throwing a few rocks, crossing in shallow areas, walking a little further–I noticed that it sometimes became difficult to hold on to the intuitive feel for which side of the creek we were on; i.e. were we on the same side of the creek as our trailer or on the same side as the main road leading up to our driveway? This was particularly true as we stood at a certain point on a little "peninsula" formed by a steep meander. From here, even I had trouble *feeling* that we were on the road side, because *from that point*, we *would* have had to cross the creek to get to the road (that is, unless we walked out of the peninsula, and the way out wasn't *visually* obvious). As we continued our walk, I occasionally asked Jesse, "Road side or trailer side?" From a navigation point of view, this helped him think more about the path the creek carved through the space we were exploring.

Later at home, I thought this would make a good puzzle (similar to Walter Wick's "String Game" in his original *Can You See What I See?*). Note that the puzzle has no *immediate* practical value. It was *motivated* by the context, and the context likely helped Jesse understand the puzzle. But I wouldn't say that the puzzle is a "real---world application" of math.

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Exploration #1: Road side or trailer side? (Creating a model)

I started by drawing a "map" with a road running straight across the top of the page, and our trailer in the bottom left. I drew a "creek" that meandered in ways a real creek likely would not; here I was more interested in the mathematics of crossings; in this sense, the diagram was a **mathematical model** that drew attention to particular aspects of creek flow while ignoring or even altering others. The hypothetical creek started on the right side of the paper and "flowed" out the left side of the paper, winding all over the page in between.

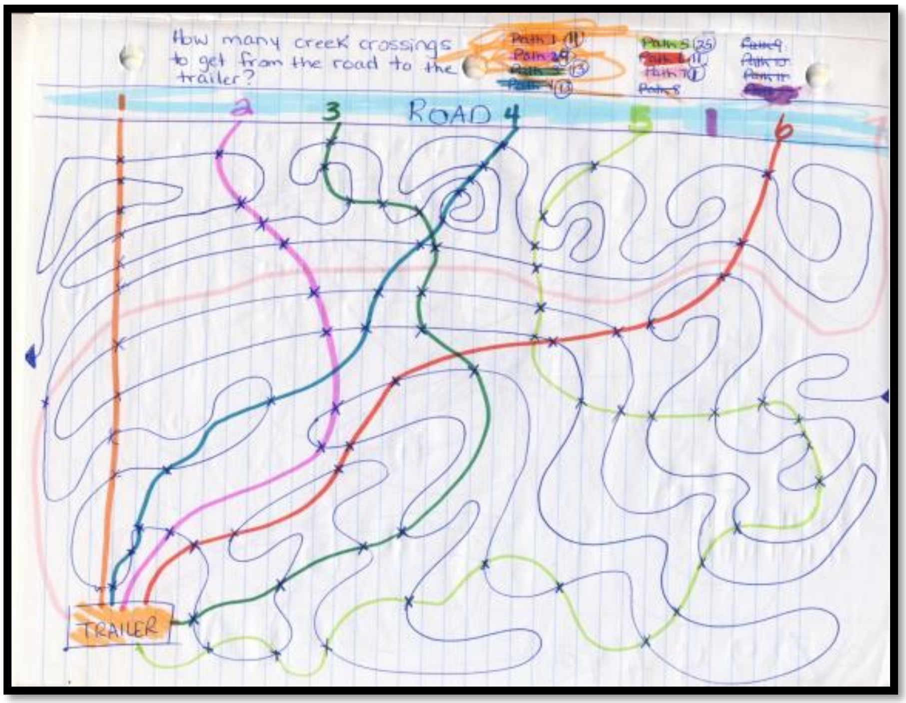
I numbered several locations within this space and asked Jesse whether they were "roadside" or "trailer side." I deliberately included a place on the page that was very close to the trailer but on the "road side" of the creek (#13) and one that was very close to the road but on the "trailer side" of the creek (#14).

He decided to draw an orange asterisk to indicate trailer side and a blue asterisk to indicate road side. Occasionally, he would notice two numbers on the same side and immediately know that they must be the same colour, but at first he did not do this systematically. Rather, he'd seek a path from each number back to the road or trailer.

Later, I suggested we colour all the parts of the map on the trailer side. He was very excited to see how the orange colour spread into all the little bumps and extensions and excited when one of his asterisks showed up in the appropriate colour zone (here, I coloured, while he gave directions; that way, the mechanics of colouring didn't get in the way of the thinking). Once coloured, the zones were obvious; this was not the case with the original image:

Exploration #2: How many crossings? (Exploring the model)

I wondered how I might draw Jesse's attention to the number of crossings one must take to get from the road to the trailer. I drew another map, and this time drew seven different paths from road to trailer, each in a different colour. I asked, "How many creek crossings to get from the road to the trailer?"



We counted:

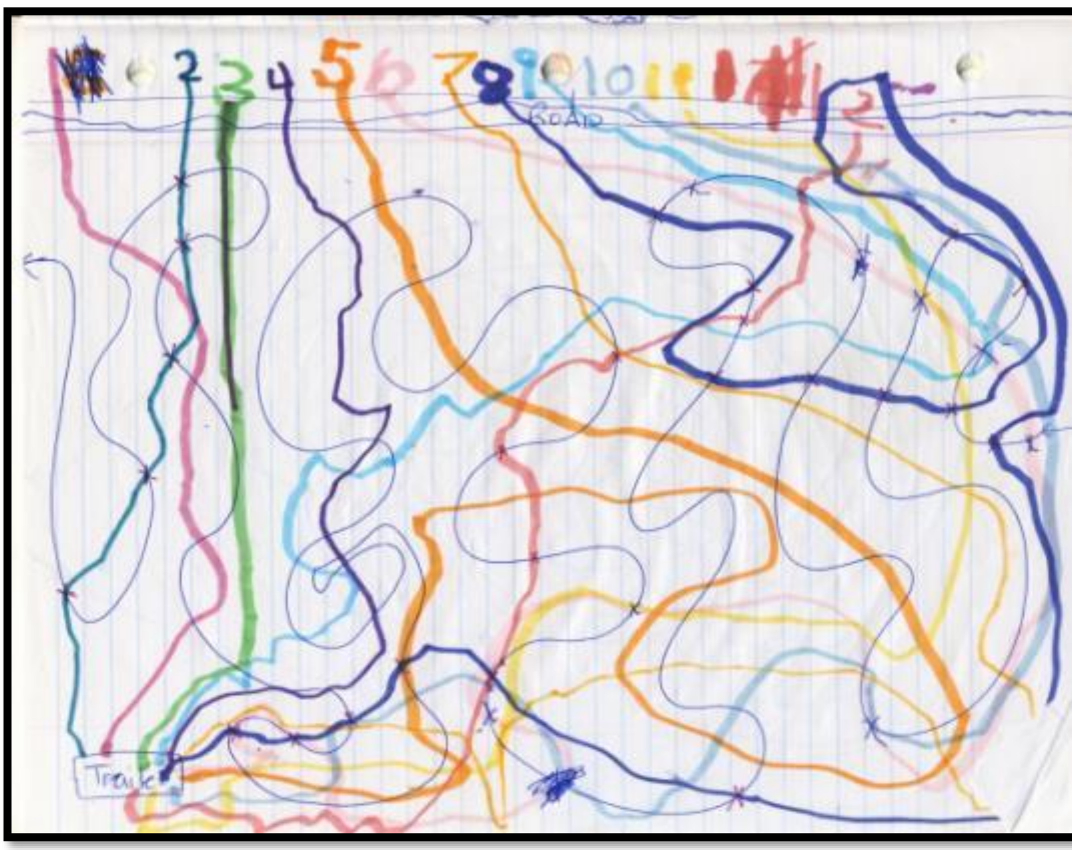
- Path 1: 11
- Path 2: 9
- Path 3: 13
- Path 4: 13
- Path 5: 25
- Path 6: 11
- Path 7: 1

In hindsight, I think I should have included more paths with fewer crossings. Path 7 was an afterthought.

Exploration #3: What are the path rules? (Refining the model)

I drew a new map, thinking I would do this, but Jesse decided he wanted to draw his own paths. Twelve crossings made the map a little crowded; where two paths cross the creek in the same spot, it becomes more difficult to keep track of crossings. But I didn't want to interrupt him---he was really into this! As he worked at creating his paths, he asked some interesting questions pertinent to our **mathematical model**:

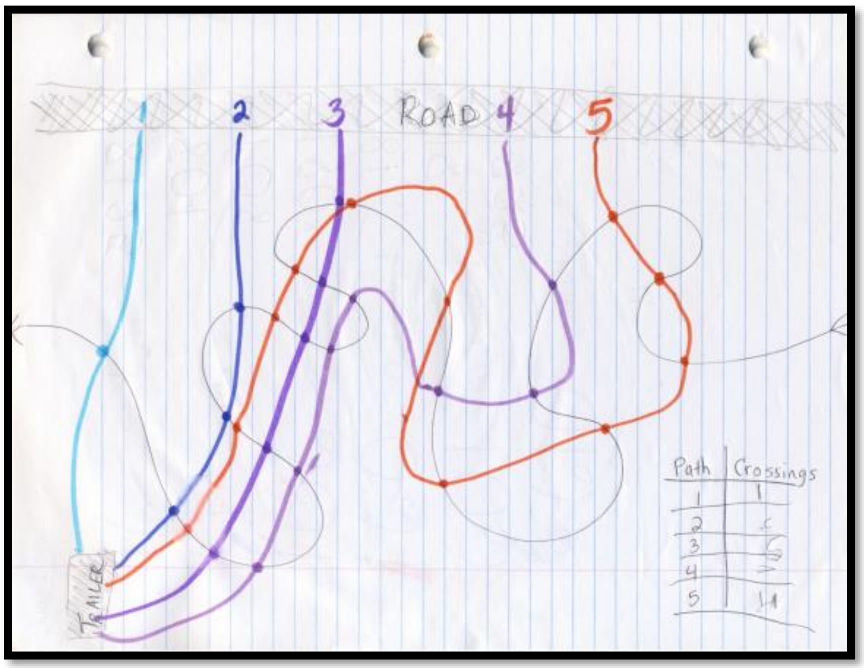
- Does it matter if the paths cross each other?
- Can a path cross over itself? He decided that he would allow both, though there's no real right answer to either question (it depends what we want to explore). We also wondered what it meant when the path line touched the creek and travelled alongside it or in it.



Exploration #4: Is the number of paths *always* odd? (Generalizing)

I noted that it seemed strange that all of the numbers we had found so far were odd. I wondered aloud whether this would **always** be the case (**generalizing**), and whether it might be *possible* to find a path with an even number of crossings. The investigation continues…. I'm not sure if he'll adopt this question as his own. If not, we'll leave it for now. If he does and runs into trouble with his crowded map, I may encourage him to draw a new one that makes *this* exploration a little more manageable. Or perhaps after he gets frustrated trying to keep track of what he wants to know on the original, I'll create a simpler one that offers a contrast.

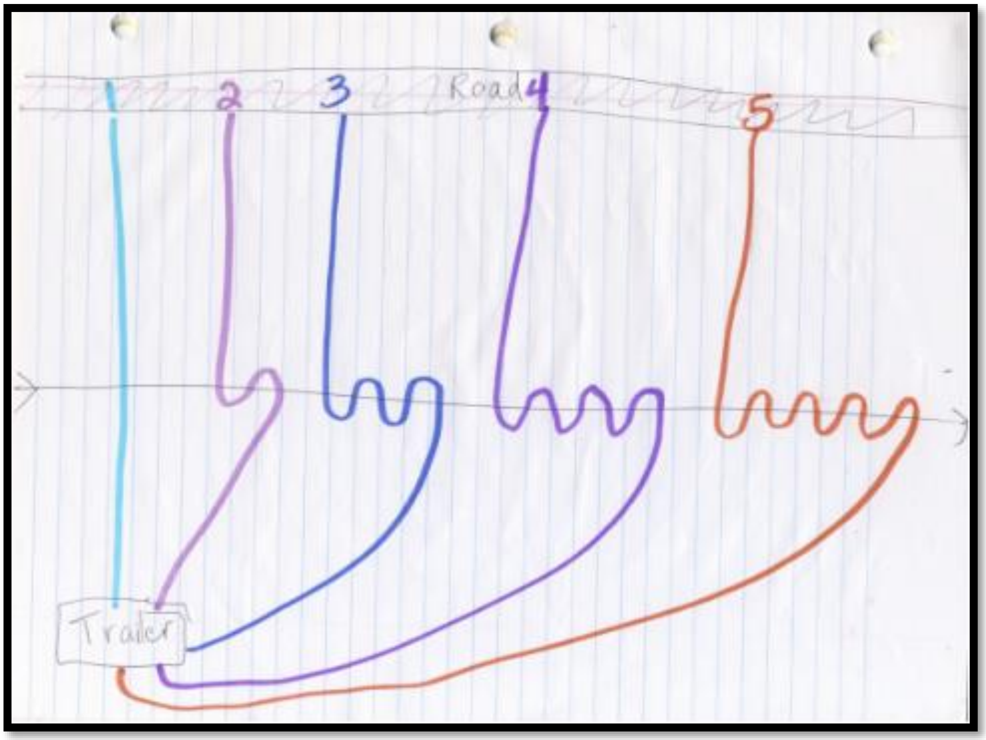
Exploration #5: Why always odd? (Simplifying the model, part 1)



Here, the line between my investigation and Jesse's is not clear. A significant part of the mathematics in this task is creating the simplified models and considering whether they (a) still accurately represent the situation we're exploring and (b) completely represent the situation we're exploring. For example, is the number of pathways always odd because of some feature that only applies to the simplified models?   Nonetheless, Jesse is still enjoying the tasks; he takes part in the parts he can, and I think there is value in him being immersed in the larger context even though he doesn't yet fully understand why I keep creating new creeks. But I'll also stay attentive for the times he says, “Hey, can we…?” or “What would happen if we…?” Those are the spaces where he continues to adopt the model as his own.

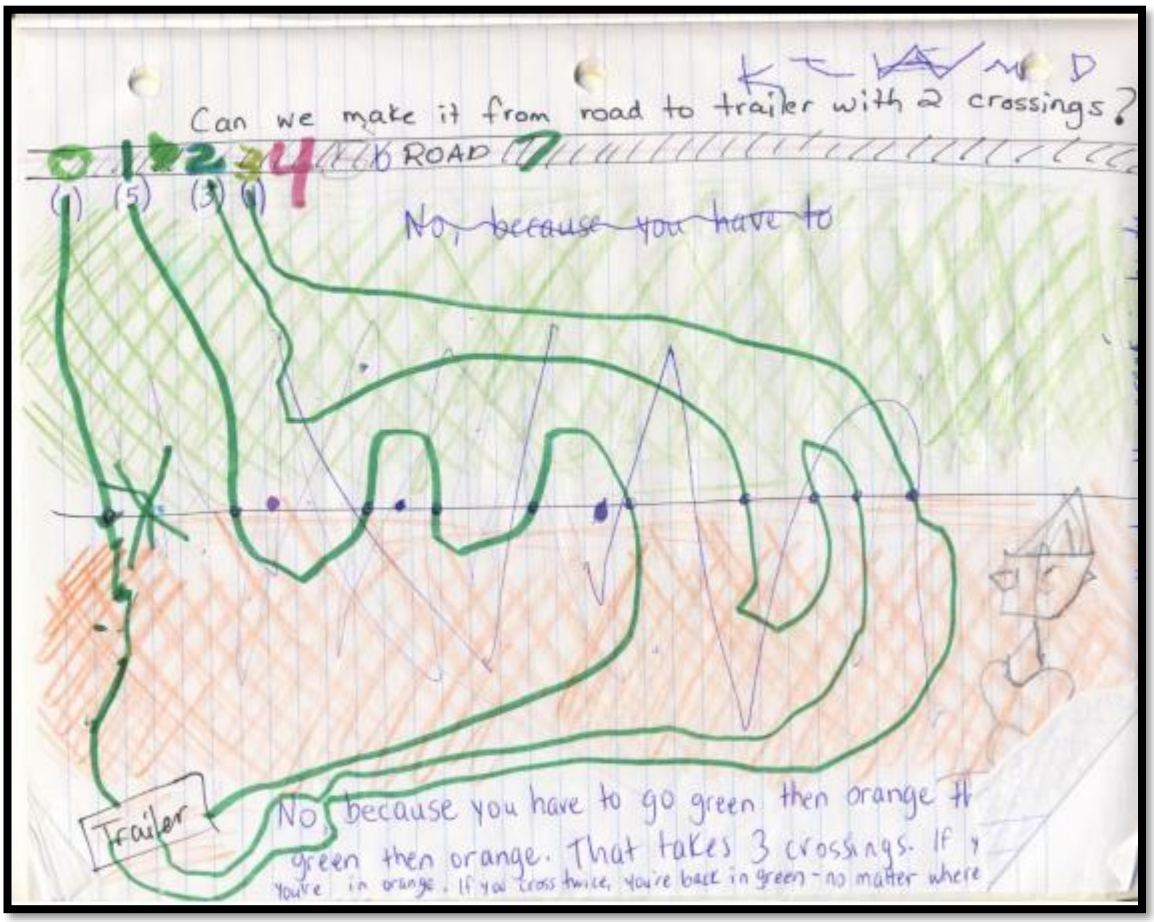
Exploration #5: Why always odd? (Simplifying the model, part 2)

When I showed Jesse the following map, he asked, “Why doesn’t the creek bend in this one?” Great question…. Again, even though he doesn’t understand my motivation in creating it, I’m wondering if it might spark some insight regarding possible numbers of crossings. I thought he’d want to extend it – here there is a more obvious pattern. But his main addition was “Path 0,” which (to me) is a variation of Path 2.

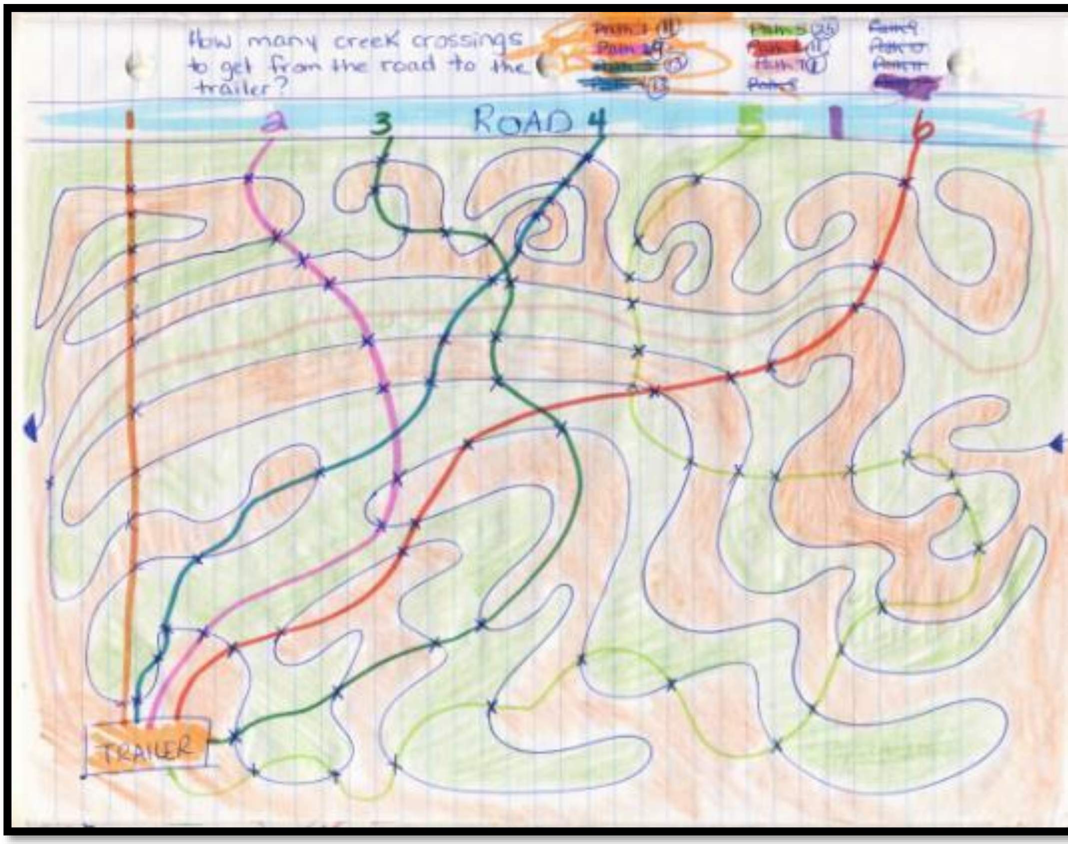


Exploration #6: Is it *possible* to create a path from road to trailer with an even number of crossings?

Here, the model becomes both (a) my attempt to bridge Jesse’s understanding with the necessity of odd crossings and (b) my attempt to make that necessity more intuitively obvious (logically, it’s already obvious to me that to get to the other side of the creek always requires a there-and-back-and-there-etc.). Jesse quickly set to work drawing paths, this time modeling them after the one in Exploration #6. For Path 0, he originally drew a forked path and noted it had 2 crossings. But he quickly recognized that such a path wouldn't allow 2 crossings unless he went over and back on one fork, resulting in 3 crossings. From here, he quickly (if a little tentatively) decided that 2 crossings was impossible:" No, because you have to go here then here then here." I suggested we colour in the green/orange spaces so that they were easier to talk about, and he then said, "No, because you have to go green, then orange, then green, then orange. That takes 3 crossings. If you cross once, you're in orange. I you cross twice, you're back in green." I asked if this was true no matter where you cross. He said yes. I asked if it also works on bendy creeks. He said yes. Not sure how deep this understanding is….

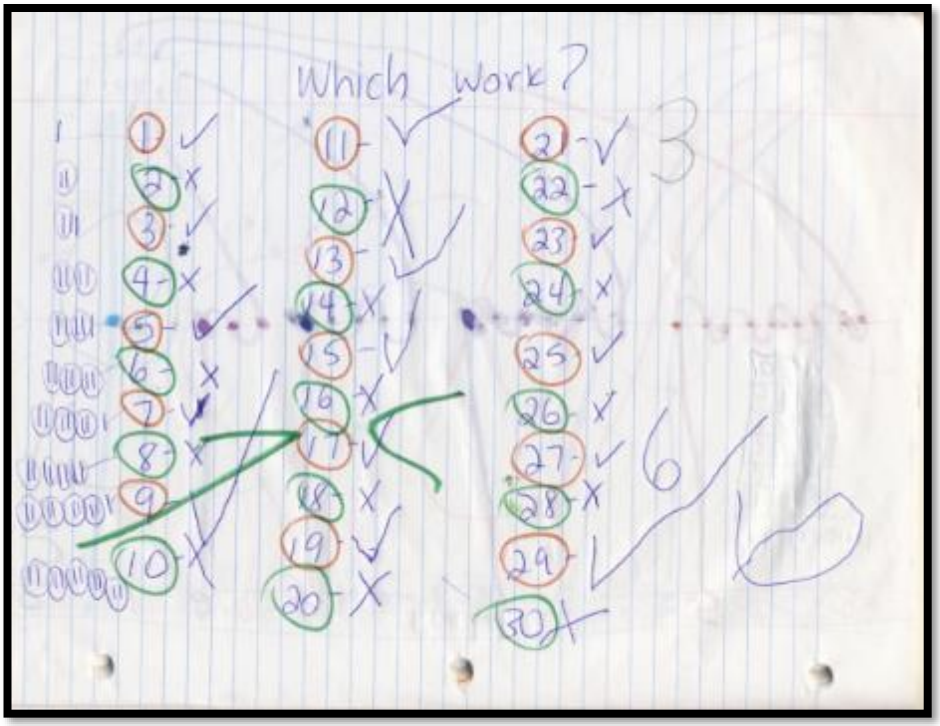


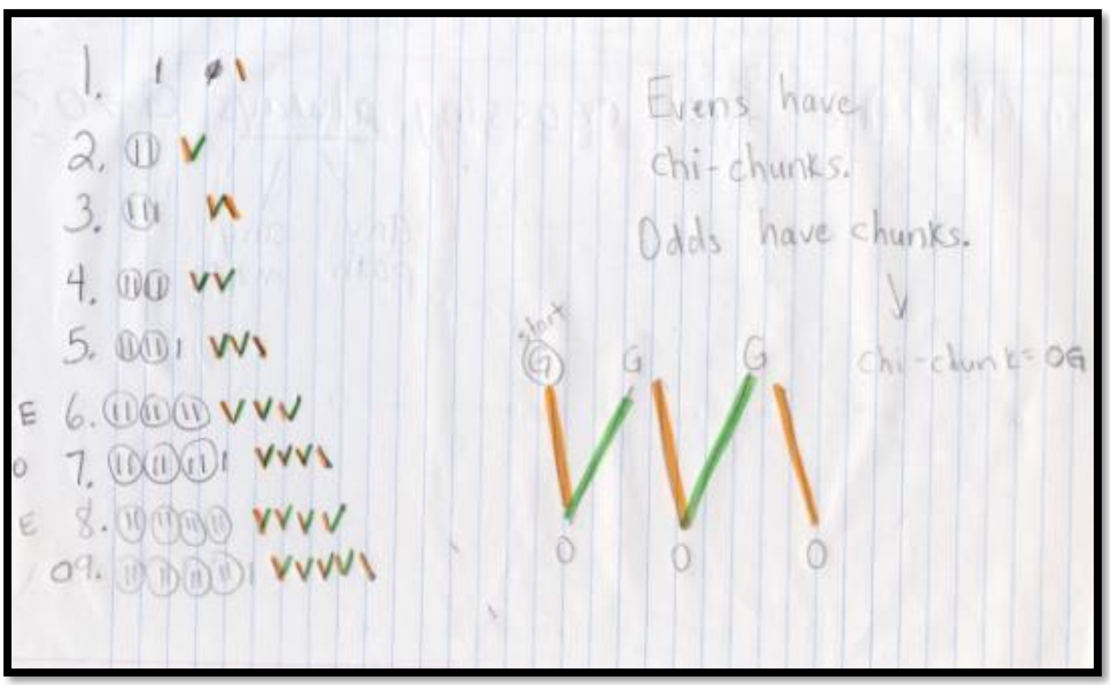
He really enjoys colouring the orange / green paths, so we went back and coloured the map for Exploration 2. Here, I asked if the first crossing is *always* G → O (on any path, on any map). We still haven't considered why even numbers other than 2 never seem to come up.



Exploration #7: What other numbers are impossible?

Now, Jesse doesn't feel the need to draw every path. He has started moving back and forth across the creek with his finger. So I made a chart for him to test various numbers. He quickly noted, "Hey, it's going in a pattern " He does not recognize evens/odds at a glance, so I tried to help make the connection between what we were doing and evens and odds; in fact, "numbers that get you back where you started from" would be a reasonable *definition* of evens in this context. There-and-back-pairs form even numbers, and one more trip creates an odd. So "working paths" always have pairs-plus-one. It strikes me that early in the exploration, *my* recognition of the significance of odd and even drove the sorts of questions I asked, but his interest wasn't yet piqued. We attended to other aspects of the model that *did* interest him, and eventually I found a question that did direct his attention to the structure of odd and even. But figuring out *wh*y the paths are all odd ended up involving using the model to *define* even and odd. By asking "why not 2?" the question was focused enough and clear enough to grab his attention, open enough to beg extension to other impossibles, and we came back to *my* original question in a roundabout sort of way. We have more work to do with this, I think….


Which work?


Evens have chi-chunks.
Odds have chunks.
Chi-chunk = OG

Exploration #8: Where *else* might we find chi-chunks?

I started referring to pairs we encountered as chi-chunks and asking whether particular collections are even (chi-chunks) or odd (chi-chunk-chis). We were out looking at leaves on the weekend, and he noted: Hey! This (see picture below) is a chi-chunk!" He pointed to leaf pairs and chanted: "Chi-chunk, chi-chunk, chi-chunk…. Chi [pointing to the top leaf]! No, it's a chi-chunk-chi!" As he was pointing to the leaf's pairs (chi-chunks), it struck me that it's kind of interesting that odd numbers can have many pairs and that their perfect symmetry can still be thrown by a single oddball. I've never really thought of odd numbers that way before.

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