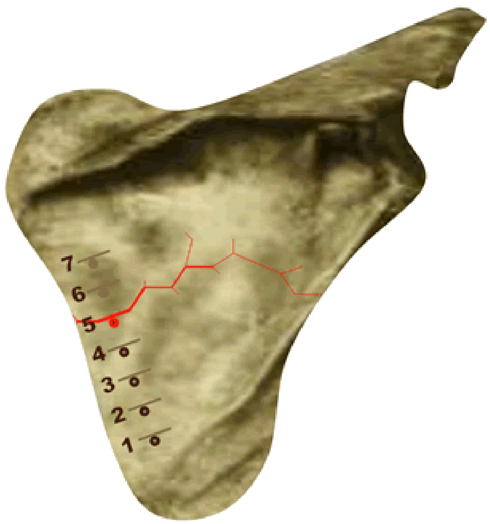
Bone Crack Divination

"Tài Wù, oh great ancestor, tell us - will the *first* day of the week will be unlucky?" Gu took the red hot poker and seared beside the first hole in the bull scapula. The bone hissed and smelled burnt, but didn't break. The king was looking at Gu, wondering if he would have to postpone his hunting trip.

Gu asked again "Tài Wù, oh great ancestor, tell us - will the *second* day of the week will be unlucky?" and plunged the red hot poker next to the second hole. Again the bone hissed but didn't break. The king looked relieved.



Gu continued till, on the 5th day of the week, there was a thunderous crack and the 5th hole split the scapula. The 5th day would be unlucky, and so the hunting party would have to be back home before the 5th.

If the probability of the scapula cracking is 1/2 each time the red hot poker is applied, what is the probability that Gu will have the bone crack for the first time on the fifth hole?

Calculus Extension:

Assuming that the probability, p, of the bull scapula splitting was the same for each day of the week, what is the probability that the hole corresponding to the 5th day of the week would be the first one to fracture the bone? For what p is this probability maximal?

History:

The earliest examples of Chinese writing date to the Shang dynasty at around 1200 BCE. The writing was on bone - usually the shell of a turtle (below left) or the scapula of a bull, cow, or other large animal (below right). They were concerned with the outcomes of divinations like the one above in which an ancestor was asked about the future. The text recorded the questions and the answers.

The Math in This Problem:

This investigation studies probability theory, which is used to draw conclusions about the likelihood of potential events through quantitative analysis. Students will apply the binomial distribution probability mass function, which generates the probability of successes in a trial, given a known probability value and number of independent experiments. After gaining an understanding of this simple distribution, students will better transition to other more intricate probability distributions.