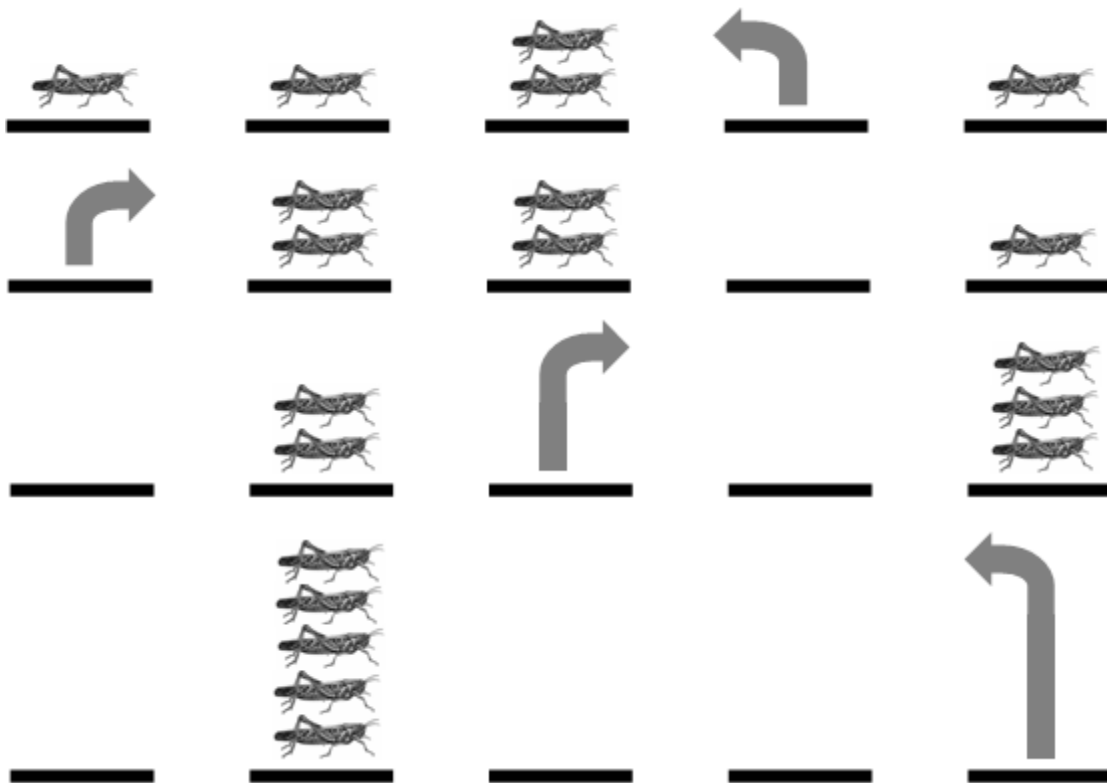


Grasshopper Jump Fest

One of the sports that grasshoppers play when they are by themselves is the jump-fest. To play, a number of grasshoppers stand on a tire tread:



Every few seconds all of the grasshoppers on one bit of the tire tread jump a number of tread-spaces equal to their number... so one grasshopper would jump one space... two grasshoppers would jump two spaces and so on... Any jump is permitted so long as it ends on top of another grasshopper and not an empty bit of tire tread.



The grasshoppers all win if they can all stack onto the same space.

Can the grasshoppers be successful when 6, 7 or 8 of them play?

How many jumps does it take to successfully complete a game?

Find a pattern of jumping that works for any number of odd grasshoppers or find an odd number for which the grasshoppers cannot win.

Find a pattern of jumping that works for any number of even grasshoppers or find an even number for which the grasshoppers cannot win. [Hint](#)

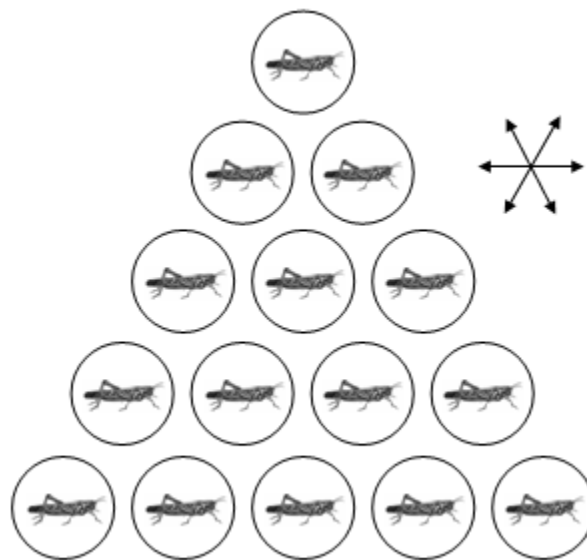


Extensions:

- When the King grasshopper plays, he just sits and expects the other grasshoppers to jump on top of him... In an 8 person game is it possible for the King to sit anywhere and still be successful?
- Is success always possible when playing with the King?
- One day a lizard suggested that 25 grasshoppers should play on a 5 by 5 square of twist-off bottle caps where the jumping can be vertical or horizontal. The lizard suggested that if they are successful, he wouldn't eat them anymore. Will the lizard have to give up his diet rich in grasshopper nutrients?



- Enjoying the festive activities, and overhearing the lizard, a slithering snake suggested that if the grasshoppers could successfully stack themselves starting from the triangle shown, then he would gladly eat the lizard for them. What became of the lizard?

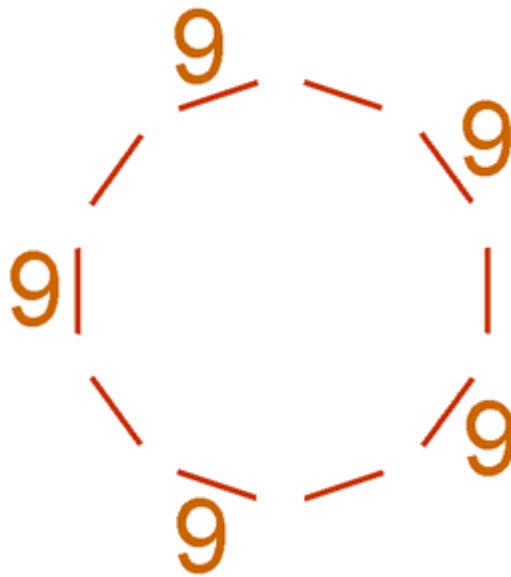


- What if one of the grasshoppers in this triangular arrangement was the King?
- Jix, an anti-social grasshopper never plays basic (tire-tread) jump-off, but laughs every time his fellow grasshoppers screw-up and fail to make a single stack. What is the fewest number of moves that Jix must wait for his fellow grasshoppers to be unable to move when 5, 6, 7 or 8 of them play?
- Jix suggests another sport: How many grasshoppers can stack onto 10 tire tread bits so that every stack can jump to an empty tread, but no stack can jump to a tread with other grasshoppers on it. For example, Jix can fit 40 grasshoppers like this:

9 8 7 6 5 _ _ 2 3 _

Fit more than these 40 grasshoppers.

- Imagine the 10 tire treads are in a circle. How many grasshoppers can stack (height less than 10) onto the treads so that every stack can jump to an empty tread, but no stack can jump to a tread with other grasshoppers on it. For example, Jix can fit 45 grasshoppers like this:



Fit more than these 45 grasshoppers.

The Math in This Problem:

This math puzzle was inspired by the stacking game DVONN, in which the objective is to accumulate pieces in stacks. Similarly, Grasshopper Jump Fest requires students to apply abstract strategy in order to develop the required grasshopper-stacking solutions.

