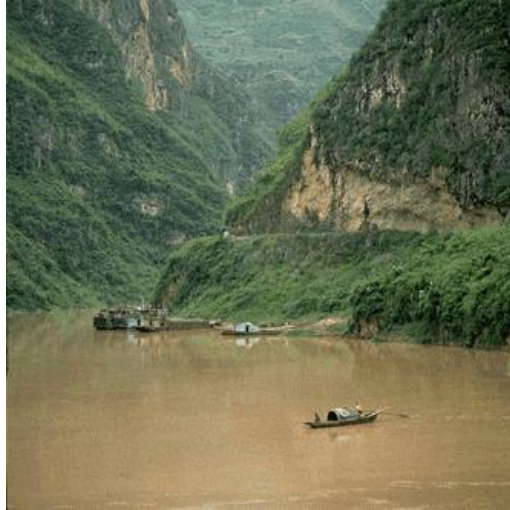


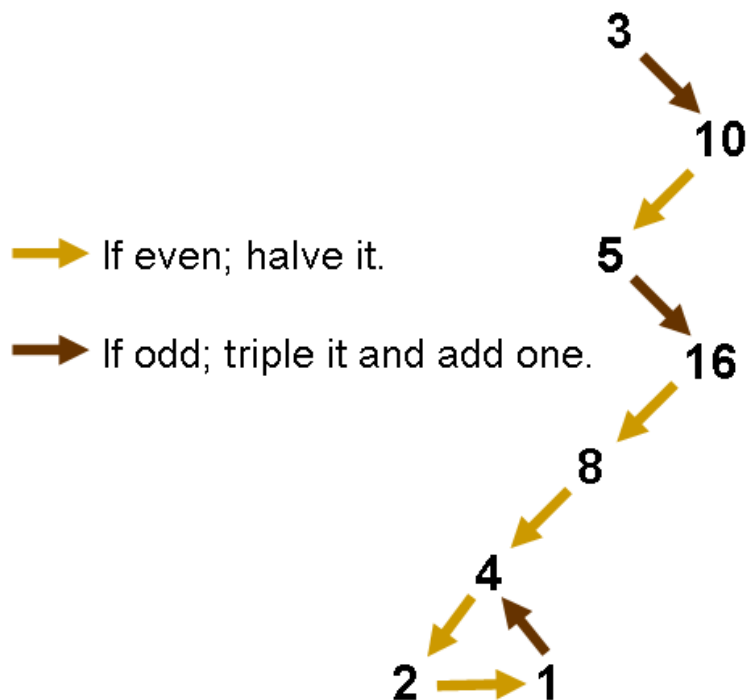
## Lothar's Number Rivers

On top of the Tibetan plateau there are two giant rivers that flow into China. The Southern one is called the Yangtze. The Northern one is called the Yellow River.



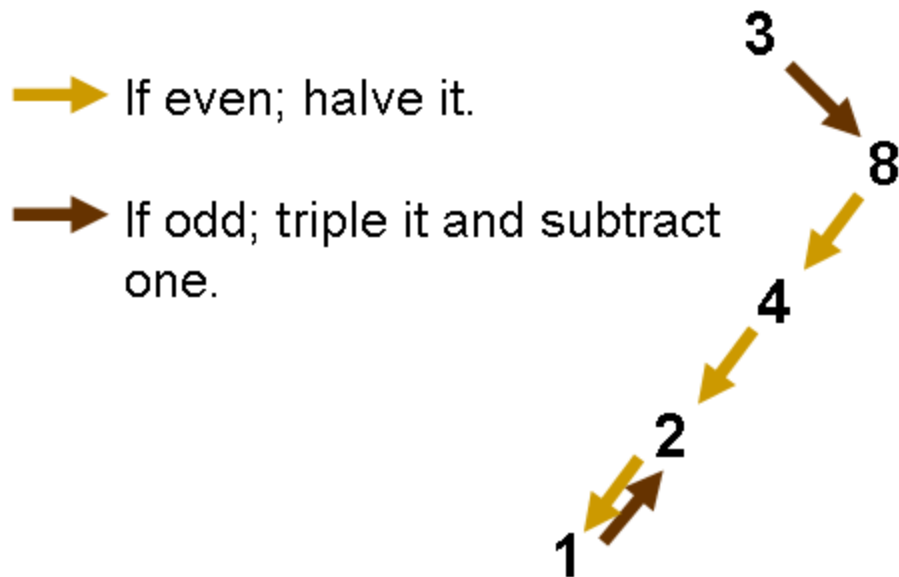
In 1937 a wandering mathematician called Lothar fell asleep on the other side of the world and dreamed of throwing positive integers into the Yellow River. He dreamed that whenever a number hit the water the mud and silt in the river started to wear it down.

If the number was even the silt (→) would cut it in half. If the number was odd the mud (→) would stick to it till it grew to one more than triple its size. This was repeated again and again until the number reached the South China Sea. Here is an example for 3:



Lothar was amazed that no matter what number he started with, it always seemed to get worn down to 1.

Next day Lothar dreamed of the Yangtze. When he threw a positive integer into this great river it also started to get worn down:



- Does the Yellow River wear down all numbers to 1?
- Does the Yangtze wear down all numbers to 1?

**\*Warning:** One of these questions is an unanswered problem in mathematics. The other is a relatively easy problem.

### Extensions:

- How does the number "25" get treated if you throw it into both rivers?
- What happens if you throw negative integers into the two rivers?
- Nick and Evan from Bishop Pinkham found that if they threw 1,000,000 into the Yellow River, that it took 152 steps to become 1.
  - Find a number larger than 1,000,000 which takes less than 152 steps to reach 1. Are there an infinite or a finite number of such numbers?
  - Find a number less than 1,000,000 which takes more than 152 steps to reach 1. Are there an infinite or a finite number of such numbers?

### The Math in This Problem:

In this puzzle, students are presented with number patterns that are formed by carrying out specified mathematical operations, depending on the values presented. Working with these various patterns, students will study their respective properties and whether or not all number sequences end in the value 1.