

# McGuire the Gathering

You are the leader of a Scottish clan; either the McGuires or the McConnells.

In preparation for war, you send out your envoys to demand clan members come to your aid. Both you and your opponent should secretly write down how many days your envoys will travel. If you send them far away, they will take longer to return, but will return with more clansmen. (Specifically, if you send them  $X$  days away, they will return with  $X$  men on each day starting on the  $X$ th day).

The winner is the first clan to have 30 clansmen more than their opponent.

For example, if McGuire sends her envoys on a 2 day journey, and McConnell sends his envoys on a 7 day journey, then the conflict follows the following course:

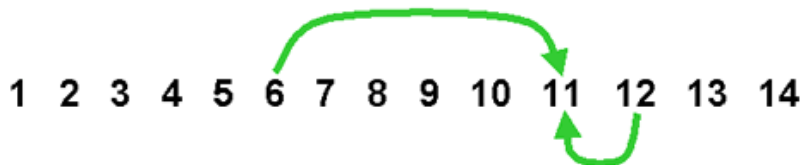
Day	McGuire Clansmen	McConnell Clansmen
1		
2	2	
3	4	
4	6	
5	8	
6	10	
7	12	7
8	14	14
9	16	21
10	18	28
11	20	35
12	22	42
13	24	49
14	26	56

The McConnell clan wins on the 14th day because there are 30 more of them.

What is a good strategy?

## Spying:

- Imagine that you have a spy that knows how your opponent will make a decision. If your spy tells you that your opponent will send his envoys out for 11 days, how many days could you send your envoy out and win? Two correct answers are 6 and 12... find all the other correct answers...



If you know what number your opponent will say, can you always win?

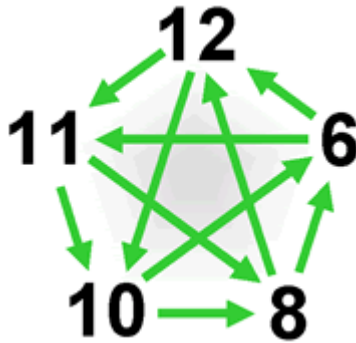
- The cycle below shows that a strategy of sending your envoys out for 6 days beats a 12 day strategy which beats a 10 day strategy which beats the 6 day strategy.



- If your spy tells you that your opponent will roll a dice and on a roll of 1 or 2 will choose a 6; on a roll of 3 or 4 will choose a 10; and on a roll of 5 or 6 will choose a 12. What numbers can you choose that will win two thirds of the time?



- If your spy tells you that your opponent will roll a dice and on a roll of 1 or 2 will choose a 6; on a roll of 3 or 4 will choose a 10; and on a roll of 5 or 6 will choose an 11. What numbers can you choose that will win two thirds of the time?



- If your spy says your opponent will choose one of the above 5 strategies with equal likelihood, what number(s) can you choose that will give you the most benefit?
- Prove that there is something unique about the group of five numbers 6, 8, 10, 11, and 12.
- Find the same type of group if the winning condition is 29 instead of 30. What about 31? Prove that all such groups contain an odd number of numbers.

- Here are the first 17 groups if you change the winning condition from 30 in the above problem. Mathematicians often collaborate, and this is a problem that I'd like people to work on and tell me anything interesting that they discover. Collaboration is useful both because it is often easy to overlook your own mistakes, and because other people can bring fresh insights.

Winning Condition	Group
1	1
2	2
3	3
4	2, 3, 4
5	3, 4, 5
6	3, 4, 5
7	4, 5, 6
8	4, 5, 6
9	3, 5, 6
10	5, 6, 7
11	4, 6, 7
12	4, 6, 7
13	5, 7, 8
14	5, 7, 8
15	5, 6, 7, 8, 9
16	4, 6, 7, 8, 9
17	6, 7, 8, 9, 10

### The Math in This Problem:

This math puzzle involves the construction and comparison of various number sequences. Progressing through this investigation will help students understand the relationship between these simple sequences and their respective linear equations, which they will learn can be used to solve a variety of useful problems.