

Twinkle Twinkle

Carl Sagan and Joseph Campbell were walking on the Milky Way last evening.

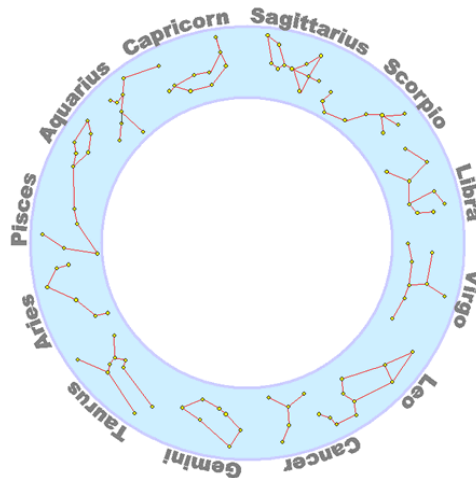
Carl turned to Joseph and said "What a lot of stars!"

"But how few constellations!" Joseph said. "Tell me Carl, how were the constellations formed?"

"Human beings are good at making patterns from what they see. Patterns help us detect trends and help us remember things too."

"So these constellations bring order and understanding to the night sky just like myths bring order and understanding to life on Earth" said Joseph.

(Carl looked like he wasn't listening as he was wrapped up in deep thought.)



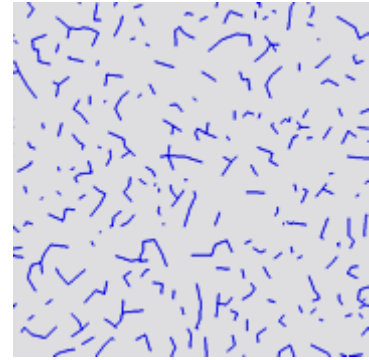
"You know Joseph," Carl suddenly said, "All of these constellations are only two dimensional snap-shots of space. If I wait for long enough or move far enough, the stars will change their relative positions so their constellations will change."

"Carl! It's just like when I used to blink my eyes open and shut during a snow storm... I could connect the dots and make stick figures of mythological creatures."



Joseph Campbell finds the mythological creature, R2D2, in the middle of a snow storm.

Carl started writing a little computer program on his palm pilot and in a few minutes he motioned to Joseph to come over. "This is an example of the type of constellations that are produced if you link up a bunch of randomly located stars to their closest neighbour. What do you think?"



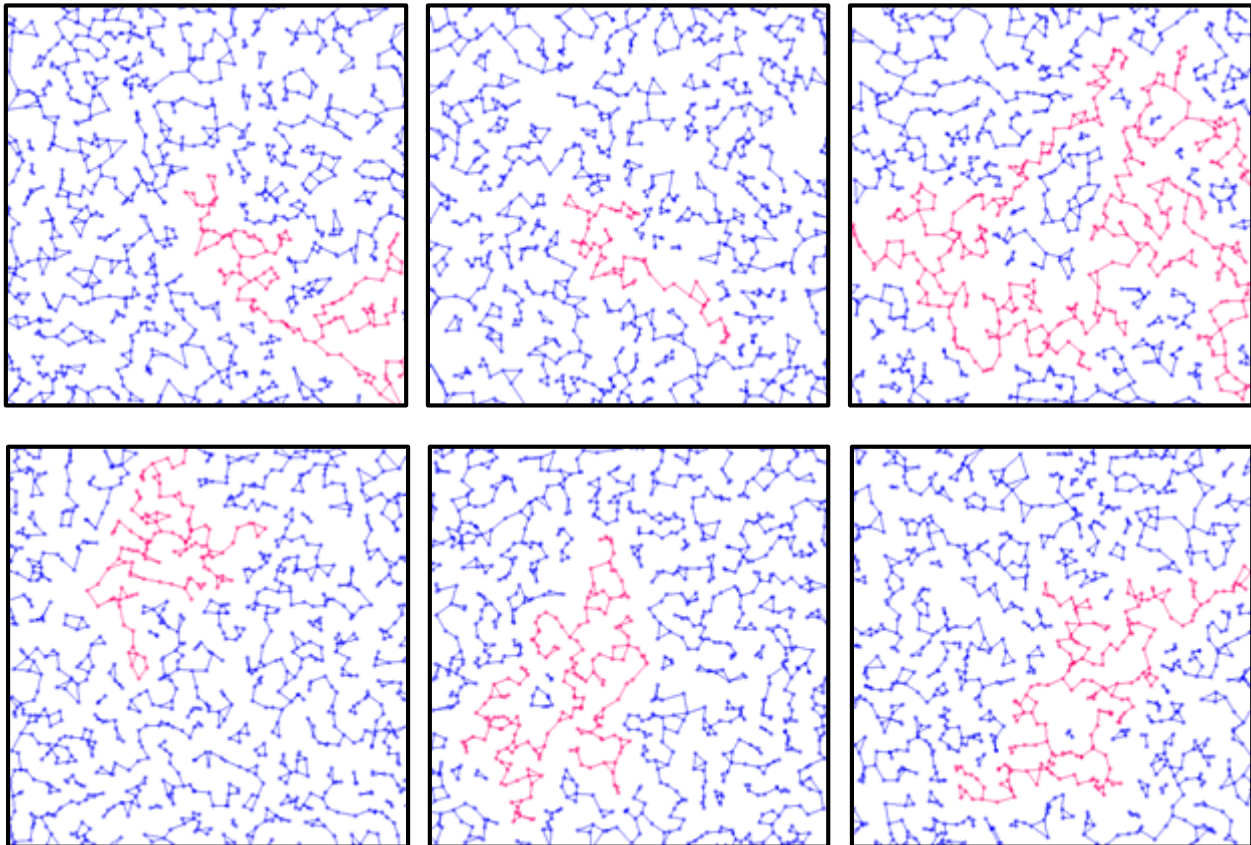
"Well Carl, honestly, those look nothing like constellations."

"Part of the reason is because (assuming there are no two distances exactly the same) there are no cycles or loops of stars."

- Is Carl correct? Are cycles and loops impossible?
- The number of stars that a star is connected to is called its valence. Assuming there are no two distances exactly the same, what is the maximum valence a star could have?
- If you start out with 15 stars, what is the greatest number of constellations you can have? Draw an example.
- What is the fewest you can have? Draw an example.

Extensions:

- Carl made a few corrections in his program, and then beckoned Joseph to look again. "This time I've connected each star to its two closest neighbours. Here are 6 randomly generated sets of stars - What do you think?"



"What's the pink?"

"That's the largest constellation in each set."

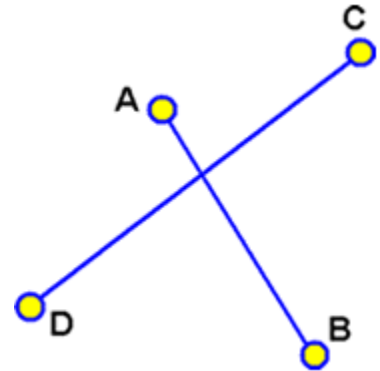
"Well it looks even more ridiculous Carl. They're enormous. No real constellation is so big. Some of them look like they cross the whole sky!"

"Yes, but you have to agree that at least loops of stars are possible."

- Is Carl correct? Draw an example.

"Maybe, but it also looks like two lines connecting stars can cross. That never happens with real constellations."

- Is Joseph correct? Is it possible that lines connecting stars can intersect like AB and CD?
- Position 5 stars so that they produce 3 intersections or show that this is impossible.
- If you start out with 10 stars, what is the greatest number of constellations you can have? Draw an example.
- What is the fewest number? Draw an example.
- Using 10 stars, create a constellation that divides up the sky into exactly 8 regions.



"Holy Fractal!" thought Carl... "I wonder if, with an infinite number of stars if the largest constellation contains an infinite number of stars... and despite this, if that cluster still contains zero percent of the total number of stars."

- Is this possible?
- Is it true? (Unanswered problem)

"Carl, why are you obsessed about constructing two dimensional constellations? Why don't you make some three dimensional constellations?"

The Math in This Problem:

Twinkle Twinkle is an exploration of astronomy, where we will investigate stars and constellations. Students will make observations about cycles and loops, as well as discover the complexity of space by using mathematics to calculate valences and constellations.