XYZ Tree

Find 3 positive integers x, y and z which satisfy this equation:

$$x^2 + y^2 + z^2 = 3xyz$$

Using this solution, find a different solution by changing one and only one of the numbers.

How many solutions does the equation have? Hint

Extensions:

- Are all triple solutions {X,Y,Z} which contain a number, N, connected to each other on the tree found here? Warning: This is an unsolved problem in mathematics.

The Math in This Problem:

This math puzzle introduces us to the Markov Diophantine equation, in which a Markov number is a positive integer x, y, or z that is part of its solution. Experimenting with various numbers, students are challenged to come up with a few of the many triple solutions to this formula, which was named after Russian Mathematician Andrey Markov.